SHORT COMMUNICATION

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The prosecutor's and defendant's Bayesian nomograms

Received: 27 September 2001 / Accepted: 23 March 2002 / Published online: 21 August 2002 © Springer-Verlag 2002

Abstract Two nomograms to calculate posterior odds and probabilities in forensic cases according to Bayes' theorem are presented.

Keywords Bayesian analysis \cdot DNA evidence \cdot Likelihood ratio

In court as in other activities involving decision-making, evidence from different sources must be interpreted in an integrated way. In the forensic field in particular, the results from the DNA typing and other special tests should be analysed along with the external evidence obtained from the police, witnesses and other experts.

Bayes' theorem provides a framework to consider both sources of evidence in a explicit way. Indeed, Bayes' formula has two components. One is related to the evidence external to the test, summarised as the prior odds or probability. The second one is the evidence supplied by the test result itself, usually summarised as the likelihood ratio of test results under the two competing hypotheses (i.e., both samples came from the same person or from different persons; the true father is the alleged father or an unrelated person). The combination of both factors allows the final (posterior) odds or probability of the situation being considered to be estimated.

Fagan published a nomogram that has become quite popular to facilitate bedside estimation of posterior probabilities in clinical medicine [1, 2]. However, it is not readily applicable to the field of legal medicine because the likelihoods to work with are usually out of range. Therefore, we have elaborated two nomograms, inspired

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PRIOR			LIKELIHOOD		POSTERIOR	
PROBABIL	ITY ODDS	RA	TIO	ODDS	PROBABILITY	
				100,000,000	to 1	
					99.9999990%	
1	to 100,000-	-	_	- 10,000,000 1	to 1	
0.001%					99,999989%	
		1				
0.01%	1 to 10,000-		- 10,000,000,000	- 1,000,000 to 1 99,999%		
		-	1,000,000,000		00.000070	
0.1%	1 to 1,000 -		- 100,000,000 -	-100,000 to 1	99,999%	
		J	- 10.000.000			
1%	1 to 100 -		- 1 000 000 -	- 10 000 to 1	00 00%	
170	1 10 100		1,000,000	10,000 10 1	33.33 /0	
		-	- 100,000			
10%	1 to 10 -		- 10,000 -	- 1,000 to 1	99.9%	
		-	- 1,000			
50%	1 to 1 -		- 100 -	- 100 to 1	99%	
		_	-10			
	10 1 1		10	40.4-4		
91%				- 10 10 1	91%	
99%	100 to 1			- 1 to 1	50%	

Fig.1 The "prosecutor's nomogram", to be used when the likelihood ratio is higher than 1

by Fagan's original that can be used in the interpretation of forensic cases. Since they apply to situations of positive (i.e., high likelihood ratios) and negative test results, respectively, we have designated them as the "prosecutor's nomogram" Fig 1 and the "defendant's nomogram" Fig 2.

The nomograms allow approximate posterior odds and probabilities from the prior odds and likelihood ratios to be easily estimated. We also feel that the explicit consideration of prior odds along the likelihood ratio may help to avoid the so-called "prosecutor's and defendant's fallacies" [3, 4]. In order to estimate posterior odds (or probability), a line is drawn from appropriate prior odds (or

PRIOR POSTERIOR LIKELIHOOD PROBABILITY ODDS ODDS PROBABILITY RATIO 100 to 1 99% 1 to 1,000 0.1% 1% 1 to 100 10 to 1 91% + 1 to 1 10% 1 to 10 50% 10 + 1 to 10 10% 50% 1 to 1 1/10 1/100 10 to 1 1/1000 1 to 100 1% 91% 1/10,000 0.1% 100 to 1 1/100,000 1 to 1,000 99% 1/1,000,000 0.01% 99.9% 1,000 to 1 1/10,000,000 -1 to 10,000 1/100,000,000 99.99% 10,000 to 1-1/1,000,000,000 + 1 to 100,000 0.001% 1 to 1,000,000 0.0001%

Fig.2 The "defendant's nomogram", to be used when the likelihood ratio is less than 1

probability) in the left axis, through the likelihood ratio in the middle axis, until the right axis, where the posterior odds can be read off. Thus, the nomograms can be used to obtain a quick estimation of how different "a priori" beliefs influence the final results. They are based on the following formulae derived from Bayes' theorem that should also be used when more precise figures are needed:

$1100_{\text{nre}} - Outo_{\text{nre}} / (11Outo_{\text{nre}})$ (1)

 $Odds_{post} = Odds_{pre} \times LR$ (2)

 $Prob_{post} = Odds_{post} / (1 + Odds_{post})$

where $Prob_{pre}$ is the prior probability, $Odds_{pre}$ the prior odds, $Prob_{post}$ the posterior probability, $Odds_{post}$ the posterior odds and LR is the likelihood ratio of the test result.

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